occurs in the combustion chamber. For example, excessive water rates at high forward velocities may quench the reacting grain while very low rates will result in superheating the liquid to produce choking in the nozzle which in turn limits the forward velocity of the vehicle. We point out that the water flow rate to the combustion chamber is not simply related to combustion efficiency and hence to thrust. The fact that the system must operate in a two phase regime to maximize the thrust should be clear from Fig. 5. At the higher launch velocities, the area in the two phase dome available for stable operation is reduced, thus limiting the choice of entry port geometry. Under these conditions the thrust developed by the vehicle can be shown to attain a maximum value as is shown in Fig. 2. If we could neglect the two-phase problem in the combustion process, Lorber's analysis would be correct.

As a concluding remark, the choice of 200 fps is quite an arbitrary choice of speed. We could have performed the same set of calculations for any arbitrary velocity. However, for the specific design criteria, e.g., size, geometry, etc., one does obtain approximately 250 fps as the maximum launch velocity. This value is again dependent upon the steam-water equilibrium diagram.

We also note an error in Eq. (2) which should read correctly: $T = (m/g_c)\,[(2Jg_c\,\Delta h + V_h^2)^{1/2} - V_h]$.

### Reference

[1] Hacker, D. S. and Lieberman, P., "Thermodynamic Performance Evaluation of Hydroduct Using a Thermite Fuel," *Journal of Hydronautics*, Vol. 3, No. 3, July 1969, pp. 139–144.

## Comment on "Analytical Prediction of the Incompressible Turbulent Boundary Layer with Arbitrary Pressure Distribution"

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SUBSEQUENT to the 1968 Stanford Conference (a confrontation?) on Computation of Turbulent Boundary Layers, improvements of the methods presented are being made by many of the participants. This paper is one of the results. The weighted-residuals (WR) method presented at the conference used an empirical $\tau$ integral lag equation, while this newer method used a mixing-length model for the $\tau$ distribution. Perhaps the authors would comment on the extent to which predictions are improved by the change of the $\tau$ model.

Much attention was given at the Stanford conference to the importance of bringing more information about the physics into the mathematical models. The WR technique is a moment method according to the morphological survey by W. C. Reynolds, and about such methods, he remarked: "information lost by the time averaging of the Navier-Stokes equations can never be regained by using multiple moments of the mean momentum equation. This information can only be regained through use of equations obtained by forming the moment before time averaging, i.e., by use of · · · the turbulent energy equation." Perhaps the authors would also care to comment on present and possible future WR methods in this regard.

Received November 10, 1969.
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## Reply by Author to A. G. Fabula

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IN reply to Fabula's comments, it is expedient to describe the relationship between the Method of Weighted Residuals (MWR) and the turbulent boundary-layer problem. The MWR is a technique for finding approximate solutions to differential equations. When applied to the turbulent boundary-layer equations there is a certain degree of freedom in the selection of 1) governing equations, 2) approximating function, 3) weighting functions, and 4) turbulence model. By proper selection of these four functions the MWR can duplicate identically many of the existing prediction methods currently in use (e.g., see Ref. 1).

The present method is similar to earlier works in terms of the form of the governing equations but differs in terms of the approximating function, weighting function, and turbulence model. The current selection of approximating and weighting functions represents a significant improvement over previous works, as reflected in the improved analytical behavior of the resulting equations.

The mixing-length model introduced in the current paper probably does not represent an improvement in predicted results per se. It was chosen because of the flexibility it lends to the method. It was recognized that existing global models, such as used in the earlier studies, restricted the solutions to second order, whereas detailed descriptions, such as the mixing-length model, impose no such limitations. Thus, in the present study, it was possible to take a cursory look at the expected convergence capability of the MWR, the results of which are discussed in the paper. A study is currently in progress to determine the reliability of various turbulence models, and it would be premature to make any conclusions regarding this at present.

The remark by W. C. Reynolds, referred to by Fabula, is certainly correct. To the author's knowledge, there is no current effort to solve such equations as the turbulent energy equation, by the MWR, in order to recover information lost by time averaging. These other equations can, in principle, be handled by the unified approach represented by the MWR—all that is required is the expenditure of effort.

### Reference

[1] Abbott, D. E. et al., "Application of the Method of Weighted Residuals to the Turbulent Boundary Layer Equations," *Proceedings of the AFOSR-IFP-Stanford 1968 Conference on Turbulent Boundary Layer Prediction*, edited by S. J. Kline, D. J. Cockrell, and M. V. Morkovin, Stanford Press, pp. 16–29 and 46–53.

Received December 10, 1969.
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